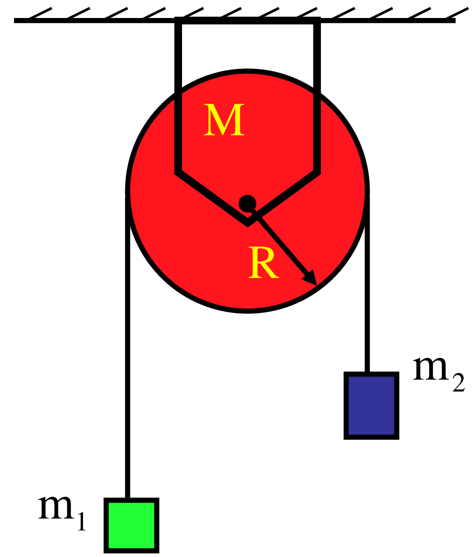
IMPORTANT NOTE (which happens to be true of all of the problems you are about to see):

Too often I will find students writing down, say, the modified conservation of energy relationship in its generic form without fitting it to the problem at hand. You will get very few points for doing that on a test. So as to be VERY CLEAR exactly what a completed governing relationship looks like, in each section of each problem of the following problem set, I have identified the appropriate governing equation, FITTED TO THE PROBLEM, by highlighting it in RED. Remembering that it is the governing equation that is all important points-wise, I would suggest you pay particular attention to how we got to that point (which is to say, I would not suggest you memorize anything—just understand how the relationships are had).

That is my friendly suggestion.

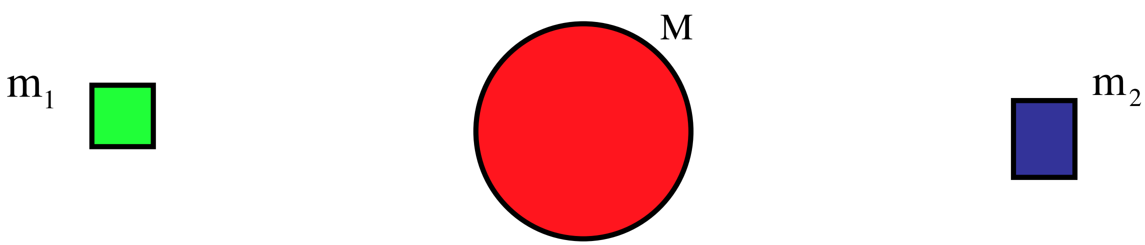
1.) A massive pulley is used in an Atwood machine. What is known is:

$$m_1,\ m_2,\ M, R, g, \text{ and } I_{cm,pully} = \frac{1}{2}MR^2$$

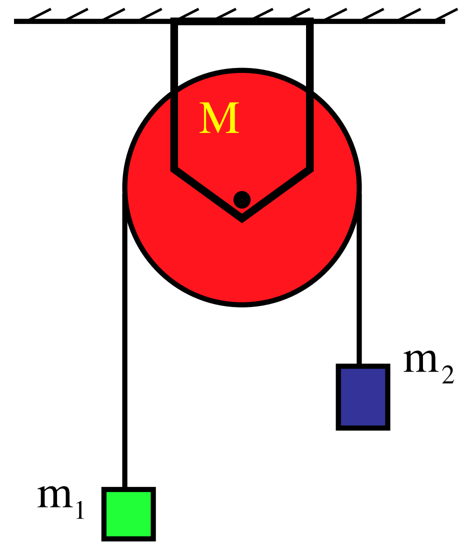



a.) Determine the *moment of inertia* about an axis perpendicular to the pulley's face and a distance R/3 from its center (this is here ONLY to give you practice using the Parallel Axis Theorem).

b.) Draw a f.b.d. for both masses and the pulley?



$m_1$, $m_2$, $M, R, g$, and $I_{cm,pully} = \frac{1}{2}MR^2$

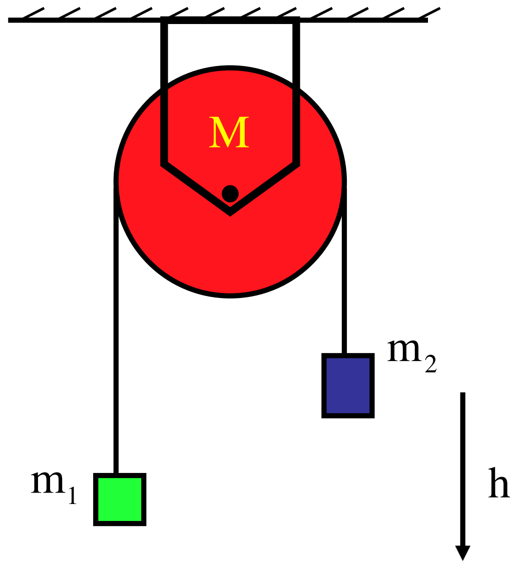



c.) What is the magnitude of the *acceleration* of the system?

d.) What is the *angular acceleration* of the pulley (include the sign)?

$$m_1,\ m_2,\ M, R, g, \text{ and } I_{cm,pully} = \frac{1}{2}MR^2$$

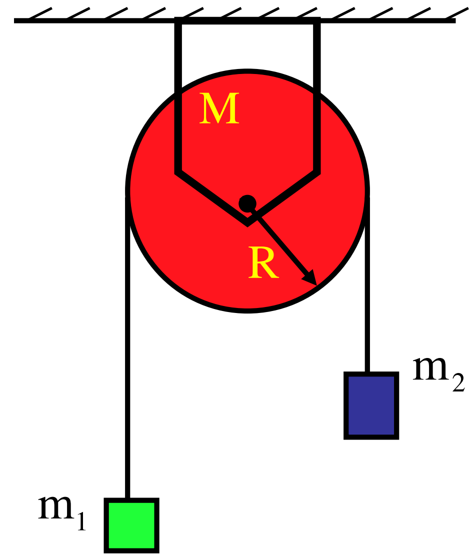
e.) The mass $m_2$ drops a distance "h." As it passes through that point, what is the magnitude of its velocity?




f.) For *Question e*, what is the *angular velocity* of the pulley?

g.) For *Question e*, what is the pulley's *angular momentum*?

1.) A massive pulley is used in an Atwood machine. What is known is: $m_1$, $m_2$, M, R, g, and $I_{cm,pully} = \frac{1}{2}MR^2$



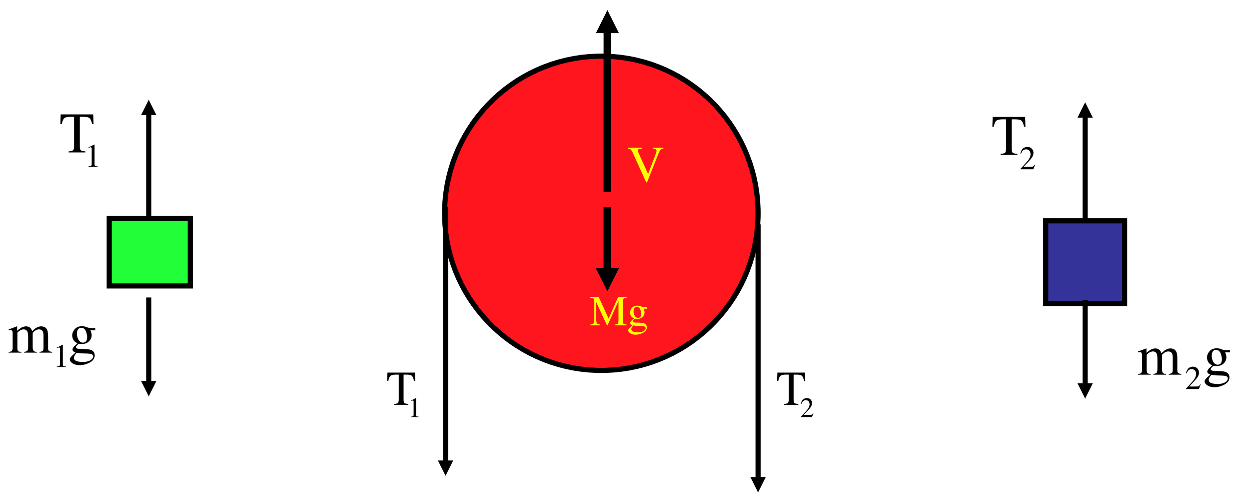

a.) Determine the *moment of inertia* about an axis perpendicular to the pulley's face and a distance R/3 from its center (this is here ONLY to give you practice using the Parallel Axis Theorem).

$$I_p = I_{cm} + m\, d^2$$

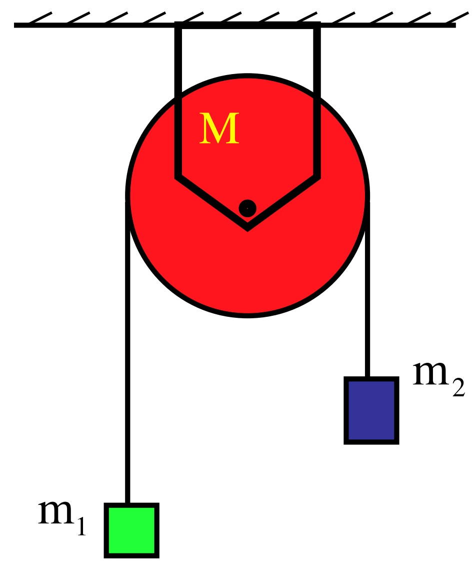
$$= \left(\frac{1}{2}MR^2\right) + M\left(\frac{R}{3}\right)^2$$

$$\Rightarrow \quad I_p = \frac{11}{18}MR^2$$

b.) Draw a f.b.d. for both masses and the pulley?



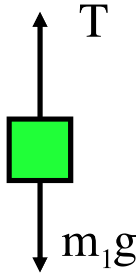
$m_1,\ m_2,\ M, R, g,$ and $I_{cm,pully} = \frac{1}{2}MR^2$




c.) What is the magnitude of the acceleration of the system? (This is the magnitude of the acceleration of *either mass*.)

To get a feel for the intricacies of this problem, let's do it first on the assumption the the pulley is NOT massive. In that case, reading from left to right, Newton's Second Law applied to each mass yields:

f.b.d. on $m_1$:

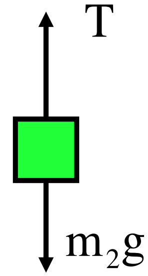



$\sum F_{1,y}$ :

$$T - m_1 g = m_1 a$$
$$\Rightarrow \quad T = m_1 g + m_1 a$$

and

f.b.d. on $m_2$:




$\sum F_{2,y}$ :

$$T - m_2 g = -m_2 a$$
$$\Rightarrow \quad T = m_2 g - m_2 a$$

$\Rightarrow$

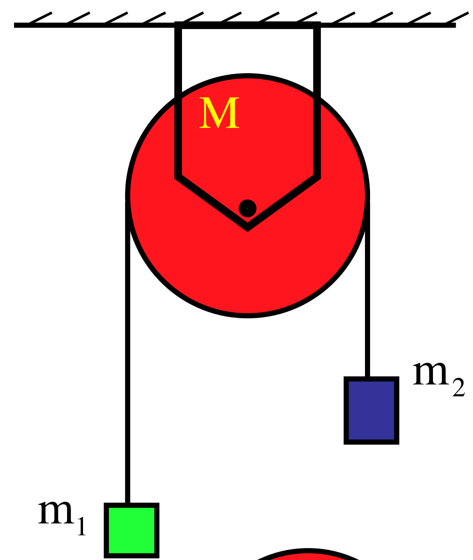
Substituting to eliminate T, we get:

$$T - m_2 g = -m_2 a$$
$$\Rightarrow \quad (m_1 g + m_1 a) - m_2 g = -m_2 a$$
$$\Rightarrow \quad a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

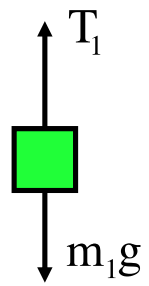
For a MASSLESS PULLEY, the tensions are the same. For a MASSIVE PULLEY, a net torque is required to make the pulley rotate. That can only come if the tension forces on either side of the pulley were DIFFERENT.

$$m_1,\ m_2,\ M, R, g, \text{ and } I_{cm,pully} = \frac{1}{2}MR^2$$

Now let's look at the situation assuming the pulley IS MASSIVE. In that case, the only difference is that the tensions are different on either side of the pulley (this has to be the case so the torque sum about the pulley's *center of mass* is not zero), and we have to deal with the fact that the massive pulley has angular acceleration. The four equations we need here are:




f.b.d. on $m_1$:



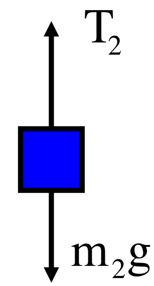

$$\sum F_{1,y}:$$

$$T_1 - m_1 g = m_1 a$$

$$\Rightarrow \quad T_1 = m_1 g + m_1 a$$

and

f.b.d. on $m_2$:



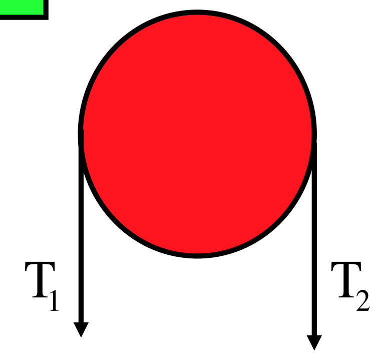

$$\sum F_{2,y}:$$

$$T_2 - m_2 g = -m_2 a$$

$$\Rightarrow \quad T_2 = m_2 g - m_2 a$$

and

(This f.b.d. selectively shows only the forces that exert torques on the pulley.)




$$\sum \Gamma_{\text{about pulley's c. of m.}}:$$

$$T_1 R - T_2 R = -I_{cm,pulley}\,\alpha$$

and

$$a_{string} = R\alpha$$

We now have four equations. Solving simultaneously, we have:

$$T_1 - m_1 g = m_1 a$$

$$\Rightarrow \boxed{T_1 = m_1 g + m_1 a \qquad \text{Equ. A}}$$

$$T_2 - m_2 g = -m_2 a$$

$$\Rightarrow \boxed{T_2 = m_2 g - m_2 a \qquad \text{Equ. B}}$$

$$\boxed{T_1 R - T_2 R = -I_{cm}\alpha \qquad \text{Equ. C}}$$

$$\boxed{a_{cm} = R\alpha \qquad \text{Equ. D}}$$

Substituting Equ. A, B and D into C, we get:

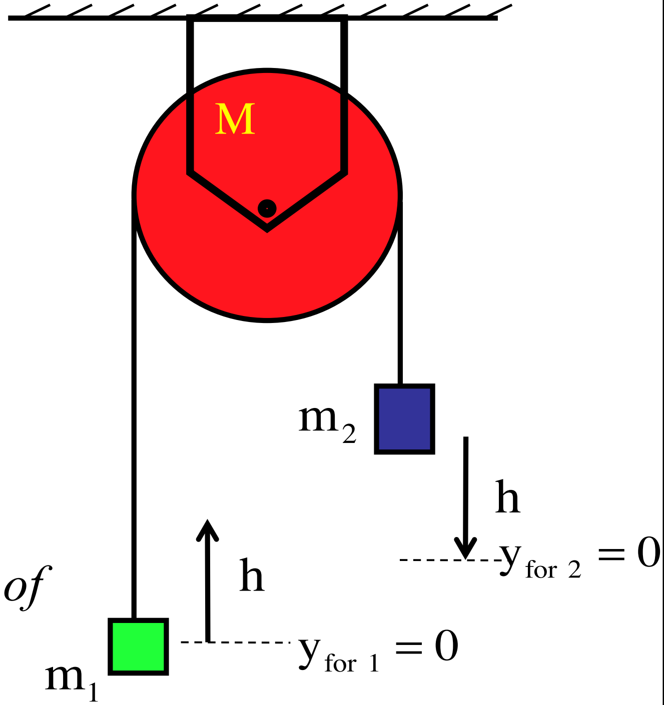
$$T_1 \quad R - \quad T_2 \quad R = - \quad I_{cm} \quad \alpha$$

$$(m_1 g + m_1 a)R - (m_2 g - m_2 a)R = -\left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$$

$$\Rightarrow \quad a = \frac{m_2 g - m_1 g}{\left(m_1 + m_2 + \frac{M}{2}\right)}$$

Note that with the exception of the presence of the "M" term, this is exactly the same relationship you got with the massless pulley analysis.

$$m_1,\ m_2,\ M, R, g, \text{ and } I_{cm,pully} = \frac{1}{2}MR^2$$




d.) What is the angular acceleration of the pulley?

This is easy. You already know "a," so you can write:

$$a_{string} = R\alpha \qquad \Rightarrow \qquad \alpha = \frac{a_{string}}{R}$$

e.) The mass $m_2$ drops a distance "h." As it passes through that point, what is the magnitude of its velocity?

This is, to some degree, where it gets exciting. The *conservation of energy* approach is useful when you had objects moving through force fields for which you know potential energy functions. The things to remember when using *conservation of energy* in combined translating and rotating settings are: a.) you need to include rotational kinetic energy if anything is moving rotationally; b.) there is no such things as rotational potential energy, so you will only need to worry about standard, translational potential energy (as you always have), and c.) it's important to remember that you CAN set the zero potential energy point for EACH BODY separately (look at sketch). With all of that in mind, let's do the problem. *Conservation of energy* yields:

$$\sum KE_1 + \sum PE_1 + \sum W_{ext} = \sum KE_2 + \sum PE_2$$

$$0 + m_2gh + 0 = \left(\frac{1}{2}m_1v_2^{\ 2} + \frac{1}{2}m_2v_2^{\ 2} + \frac{1}{2}I_{cm,pulley}\omega_2^{\ 2}\right) + m_1gh$$

As $v_{string} = v_{pulley's\ edge} = R\omega$, and as the moment of inertia for a pulley can be approximated as that of a disk (i.e., $\frac{1}{2}MR^2$), we can write:

$$\sum KE_1 + \sum PE_1 + \sum W_{ext} = \sum KE_2 + \sum PE_2$$

$$0 + m_2gh + 0 = \left(\frac{1}{2}m_1v_2^{\,2} + \frac{1}{2}m_2v_2^{\,2} + \frac{1}{2}\, I_{cm,pulley}\, \omega_2^{\,2}\right) + m_1gh$$

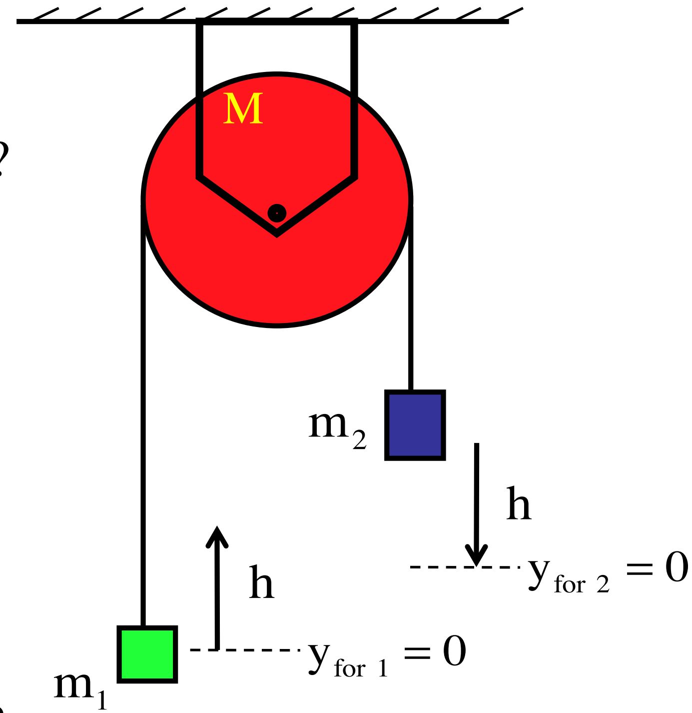
$$0 + m_2gh + 0 = \left(\frac{1}{2}m_1v_2^{\,2} + \frac{1}{2}m_2v_2^{\,2} + \frac{1}{2}\left(\frac{1}{2}M\cancel{R^2}\right)\left(\frac{v_2}{\cancel{R}}\right)^2\right) + m_1gh$$

$$\Rightarrow m_2gh = \left(\frac{1}{2}m_1v_2^{\,2} + \frac{1}{2}m_2v_2^{\,2} + \frac{1}{4}Mv_2^{\,2}\right) + m_1gh$$

$$\Rightarrow v = \sqrt{\frac{2\left(m_2gh - m_1gh\right)}{\left(m_1 + m_2 + \frac{1}{2}M\right)}}$$

Note that on a test, the second line (the one in red) would be worth 90% of the points in this part. Don't forget, the algebra is worth very little.

$$m_1, m_2, M, R, g, \text{ and } I_{cm,pully} = \frac{1}{2}MR^2$$




f.) For *Question e*, what is the *angular velocity* of the pulley?

This is easy. As you already know v, you can write:

$$v_{string} = R\omega \quad \Rightarrow \quad \omega = \frac{v_{string}}{R}$$

$$\Rightarrow \quad \omega = \frac{v}{R} = \frac{1}{R}\sqrt{\frac{2(m_2gh - m_1gh)}{\left(m_1 + m_2 + \frac{1}{2}M\right)}}$$

g.) For *Question* e, what is the pulley's *angular momentum*?

A rotating body's angular momentum is simply the *moment of inertia* of the body times its *angular velocity*. That is:

$$L = \quad I \qquad\qquad \omega$$

$$= \left(\frac{1}{2}MR^2\right)\left[\left(\frac{1}{R}\right)\sqrt{\frac{2(m_2gh - m_1gh)}{\left(m_1 + m_2 + \frac{1}{2}M\right)}}\right]$$